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FEASIBILITY OF PRODUCING PRESCRIBED DISTRIBUTION OF TEMPERATURE GRADIENT WITH ARRAY OF SUBMERGED JETS

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It is demonstrated that it is feasible to produce a region with a linear transverse temperature profile with the necessary temperature gradient along the jet height.

Considerable interest of researchers has been attracted to the feasibility of producing rather large transverse density gradients by means of devices utilizing submerged jets with intricate initial and velocity profiles [1-5].

The results of several studies [6, 7] suggest that using the method of the equivalent problem in theory of heat conduction is expedient even in the design of non-self-adjoint turbulent liquid or gas jets.

We will write the equations of a turbulent boundary layer as

$$\rho U \frac{\partial U}{\partial t} + \rho U \frac{\partial U}{\partial y} = \frac{\partial}{\partial y} \tau_{\text{т}}, \quad \frac{\partial}{\partial z} (\rho U) + \frac{\partial}{\partial y} (\rho U) = 0,$$

$$\rho U c_p \frac{\partial}{\partial z} \Delta T + \rho U c_p \frac{\partial}{\partial y} \Delta T = \frac{\partial}{\partial y} q_{\text{т}}, \quad \Delta T = T - T_0. \tag{1}$$

The change of variables

$$\xi = \xi(x, y); \quad \eta = \eta(x, y); \quad \xi_{\text{т}} = \xi_{\text{т}}(z, y); \quad \eta_{\text{т}} = \eta_{\text{т}}(z, y) \tag{2}$$

reduces system (1) to the system of linear equations

$$\frac{\partial}{\partial \xi} (\rho U^2) = \frac{\partial^2}{\partial \eta^2} (\rho U^2);$$

$$\frac{\partial}{\partial \xi_{\text{т}}} (\rho U c_p \Delta T) = \frac{\partial^2}{\partial \eta_{\text{т}}^2} (\rho c_p U \Delta T). \tag{3}$$

A solution of the equations of a boundary layer for jet-type sources by the method of an

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TABLE 1. Experimental Values of Jet Temperature at Exit from Various Sections in Various Modes of Flow

Mode of flow	T_1	T_2	T_3	T_4	T_5	T_6
I	50	75	77	80	55	5
II	55	90	85	75	55	5
III	60	100	95	75	60	5
IV	65	110	105	90	65	5
V	75	120	147	95	75	5
VI	80	145	145	115	90	5
VII	85	160	160	130	95	5

asymptotic layer and the method of the equivalent problem in the theory of heat conduction will yield the relation between old and new variables

$$\xi = cz^2; \quad \eta = y. \tag{4}$$

Such a relation was used in other studies [1, 4] pertaining to a plane turbulent jet of finite size with an initial velocity profile in the form of a step and a plane jet in a jet stream. In that case the constant c was determined experimentally and the agreement between theoretical and experimental data was found to be close. The heat problem [1, 4] was solved analogously, with the change of independent variables

$$\xi_T = c_T z^2; \quad \eta_T = y. \tag{5}$$

Using the method of the equivalent problem in the theory of heat conduction, we will seek the solution to Eqs. (3) for the initial and boundary conditions

$$\begin{gathered}
\text{at} \quad 1 \geqslant \bar{y} \geqslant \frac{2}{3} \quad \Delta T = \Delta T_1; \quad \frac{2}{3} > \bar{y} \geqslant \frac{1}{3} \quad \Delta T = \Delta T_2; \\
\frac{1}{3} > \bar{y} \geqslant 0 \quad \Delta T = \Delta T_3; \quad 0 > \bar{y} \geqslant -\frac{1}{3} \quad \Delta T = \Delta T_4; \\
-\frac{1}{3} > \bar{y} \geqslant -\frac{2}{3} \quad \Delta T = \Delta T_5; \quad -\frac{2}{3} > \bar{y} \geqslant -1 \quad \Delta T = \Delta T_6; \\
-1 > \bar{y} \geqslant 1 \quad U = U_0.
\end{gathered} \tag{6}$$

The general solution to Eqs. (3) for the initial conditions (6) is [8]

$$\frac{U}{U_0} = \frac{1}{\sqrt{2}} \left[\operatorname{erf}\left(\frac{\bar{y}+1}{2\sqrt{\bar{\xi}}} \right) - \operatorname{erf}\left(\frac{\bar{y}-1}{2\sqrt{\bar{\xi}}} \right) \right]; \tag{7}$$

$$\begin{aligned}
\frac{\Delta T}{\Delta T_1} = \frac{1}{2} \frac{U_0}{U \Delta T} \Bigg\{ & \Delta T_6 \left[\operatorname{erf}\left(\frac{-1-\bar{y}}{2\sqrt{\bar{\xi}}} \right) - \operatorname{erf}\left(\frac{-\frac{2}{3}-\bar{y}}{2\sqrt{\bar{\xi}}} \right) \right] + \\
& + \Delta T_5 \left[\operatorname{erf}\left(\frac{-\frac{2}{3}-\bar{y}}{2\sqrt{\bar{\xi}}} \right) - \operatorname{erf}\left(\frac{-\frac{1}{3}-\bar{y}}{2\sqrt{\bar{\xi}}} \right) \right] + \Delta T_4 \operatorname{erf}\left(\frac{-\frac{1}{3}-\bar{y}}{2\sqrt{\bar{\xi}}} \right) + \Delta T_3 \operatorname{erf}\left(\frac{\frac{1}{3}-\bar{y}}{2\sqrt{\bar{\xi}}} \right) + \\
& + \Delta T_2 \left[\operatorname{erf}\left(\frac{\frac{2}{3}-\bar{y}}{2\sqrt{\bar{\xi}}} \right) - \operatorname{erf}\left(\frac{\frac{1}{3}-y}{2\sqrt{\bar{\xi}}} \right) \right] + \Delta T_1 \left[\operatorname{erf}\left(\frac{1-\bar{y}}{2\sqrt{\bar{\xi}}} \right) - \operatorname{erf}\left(\frac{\frac{2}{3}-\bar{y}}{2\sqrt{\bar{\xi}}} \right) \right] \Bigg\},
\end{aligned} \tag{8}$$

with $\bar{y} = \frac{y}{h}$; $\bar{\xi} = \frac{\xi}{h}$; and h denoting the half-height of a jet.

The sought functions were evaluated according to expressions (7), (8) for various initial conditions indicated in Table 1. By way of algebraic recalculation according to relations (4) and (5), one can obtain the real velocity and temperature profiles upon introduction of coefficients c and c_T. These coefficients were determined through minimization of functionals

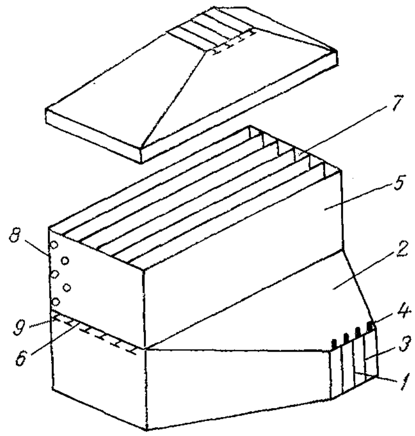

Fig. 1. Basic schematic diagram of experimental apparatus.

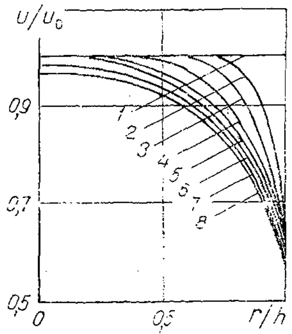

Fig. 2

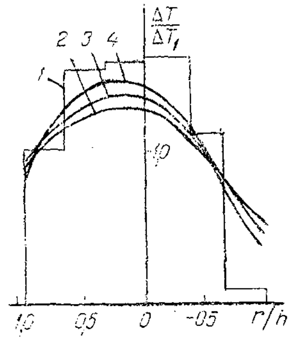

Fig. 3

Fig. 2. Velocity profiles at various heights along jet: 1) z/h = 0; 2) 0.286; 3) 0.572; 4) 0.858; 5) 1.14; 6) 1.43; 7) 1.715; 8) 2.0.

Fig. 3. Profiles of excess temperature at various heights along jet: 1) z/h = 0; 2) 1.44; 3) 1.715; 4) 2.0.

$$F(c) = \sum_i (U^i_{\text{theo}} - U^i_{\text{exp}})^2 \text{ and } F(c_T) = \sum (\Delta T^i_{\text{theo}} - \Delta T^i_{\text{exp}})^2,$$

with U^i_{theo} and U^i_{exp} denoting, respectively, theoretical and experimental values of the velocity, and ΔT^i_{theo} and ΔT^i_{exp} denoting, respectively, theoretical and experimental values of the excess temperature along the height of a submerged jet. These calculations were made on a computer. The values $c = 0.03$ and $c_T = -0.0375$ were obtained as a result.

An experimental apparatus with general configuration as shown in Fig. 1 has been built on the basis of an analysis of theoretical results. Here air was fed in through the receiving window 1 at the inlet to the distributor 2. Uniform flow of air through the entire 1200-mm-long apparatus was achieved by means of the guiding slider 3 and the regulating slider 4 in the distributor. The oven 5 was mounted above the distributor 2 and, for the purpose of distributing the air more uniformly over the cross section, a metal grid 6 with an approximately 80% porosity factor was installed across the channel. Asbestos–cement plates 7 subdivided the oven into six compartments with identical cross sections. In each compartment were installed electric heaters 8 in a crisscross pattern, these heaters being of the NSVT-250 thermal electric type. The heaters in each compartment were connected in parallel and energized from an RNO-250 transformer, their total power amounting to 6 kW. The velocity of the air stream was regulated by means of the slider 9.

Air was blown in by a Ts-13-50 fan No. 5 with a capacity of $1.2 \cdot 10^4$ m^3/h and a pressure head of 300 mm H_2O. The air flow rate was regulated by means of a slider. Air was sucked out by a U13-50 fan No. 8 with a maximum capacity of $7.2 \cdot 10^4$ m^3/h and a pressure head of 300 mm H_2O.

A uniform air velocity over the entire cross section of a jet leaving the oven was established by sliders 4, 9 and measured with a "Disa" thermoanemometer. The velocity distribution over the jet cross section was found to become uniform at a height of 3 cm, the magnitude of the velocity reaching a maximum of 4 m/sec along the height of the jet. A typical velocity distribution over the jet height agreed with the theoretical one shown in Fig. 2.

The temperature distribution over the jet height was measured with a precalibrated copper–constantan thermocouple and a UPIP-60 M potentiometer. The imprecision of thermocouple location at any point along the stream did not exceed 0.1 mm. The dependence of the temperature on the height along the jet was determined for the purpose of evaluating the coefficients c and c_T, both needed for solving Eq. (3).

The graphs in Figs. 2, 3 depict theoretical profiles of velocity and excess temperature at various heights along a jet with various initial conditions.

According to the data in Fig. 3, the jet temperature varied insignificantly within the $0 \leqslant z/h \leqslant 2$ range of heights along the jet, and the transverse temperature gradient was almost uniform over the $0 > r/h > -1$ range of the cross section. An analysis of the curves in Figs. 2, 3 leads, furthermore, to the conclusion that by varying the initial temperature distributions over the jet cross section at the discharge orifice, one can regulate the temperature gradient at a given height as well as the size of the region with a linear transverse temperature distribution.

NOTATION

U, vertical velocity component; V, longitudinal velocity component; T, temperature; τ, time; q_T, thermal flux; y, transverse coordinate; z, longitudinal coordinate; c_p, specific heat; and h, jet half-width.

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